INTERACTIONS BETWEEN AN ELECTRON STREAM AND AN ARC DISCHARGE PLASMA

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Analyses have been carried out on the interaction of an electron stream with a plasma which indicate that a velocity or density modulation applied to the electron stream will be amplified exponentially with distance when the electron stream passes through a plasma column. Amplification should occur for modulation frequencies below the plasma frequency, reach a peak at or near the plasma frequency, and extend slightly into the region of modulation frequency greater than the plasma frequency, depending upon collisions and the velocity distribution present among the electrons of both media.

An apparatus has been constructed which has demonstrated the existence of these growing waves in a plasma and which closely verifies the predictions of the analysis. Net amplification of 10 db has been observed between input and output. This apparatus also permits accurate observations to be made on the internal plasma density of an arc column with relatively high precision. At the same time, measurements have been made on the arc column to determine its plasma density by other methods. In particular, the cavity perturbation technique has been applied and is found to be in good agreement with the growing wave measurements. Plasma density measurements have also been made utilizing electromechanical modes of wave propagation on plasma columns. Also the reflection of microwaves from the arc column has been investigated.

INTRODUCTION

Recent work in our laboratory has been concerned with the experimental verification of plasma interaction theories which predict spatially growing waves in a plasma which is traversed by a beam of fast charged particles. The growing wave results from the excitation of oscillations of the plasma electrons by the beam and the interaction of the oscillating electrons back on the beam. Previous methods employed to excite plasma oscillations with a directed beam have been described in the literature as unsuccessful. This paper describes results of a new experiment in which the beam is modulated by a microwave signal before it interacts with the plasma, and is then demodulated upon leaving the plasma interaction region. A schematic is shown in Fig. 1.

A 400-volt electron beam is passed along the axis of the positive column of a mercury arc discharge 1 cm in diameter at a controlled pressure of $2 \times 10^{-3}$ mm Hg, the Hg well being held at a constant temperature of 26, 80° C. Before entering the plasma interaction region the electron beam is modulated over the range 1, to 4.0 kilomocycles (5 band) with a short helix which propagates a

*Work supported by the Office of Naval Research

Presented at the Symposium on Electronic Waveguides, Polytechnic Institute of Brooklyn, April 8-10, 1958.
slow electromagnetic wave with about the same velocity as the electron beam. The beam then passes along the axis of the arc discharge for a distance of five centimeters. Upon leaving the plasma interaction region the modulated beam induces a microwave signal on the second helix which is then coupled into the output waveguide. The following analysis will indicate that when the modulation frequency coincides approximately with the plasma oscillation frequency of the arc discharge, a strong interaction occurs and the output signal level increases sharply due to the growth in signal level along the beam.

Analysis

It has been shown by Bohm and Gross\(^3\) that the one-dimensional small-signal dispersion relation of an electron beam in a plasma is given by

\[
1 = \frac{\omega_b^2}{(\omega - \gamma u_b)^2} + \frac{\omega_p^2}{\omega(\omega - iv)} \int \frac{\varepsilon_0(u) \lambda u}{1 - \frac{\gamma \cdot u}{\lambda^2 + iv}} \]  

(1)
where all waves are assumed to vary as $e^{i(\omega t - \gamma z)}$, i.e., plane waves; \(u_b\) is electron beam velocity in the z direction; \(\gamma\) the propagation constant in the z direction; and \(\nu_c\) is the collision frequency of the plasma electrons. \(\omega_p^2\) and \(\omega_b^2\) are the plasma frequencies of the arc plasma and beam, respectively, the former being defined by

$$\omega_p^2 = \frac{\rho e^2}{m\varepsilon_0}$$

\(f_0(u)\) is the distribution function of the plasma electrons and when multiplied by \(du\) represents the fraction of all the plasma electrons in the velocity class \(u\) to \(u + du\).

The electrons in the positive column of an arc discharge have a Maxwellian distribution of energies given by

$$f_0(u) = \left[ \frac{3}{2\pi w^2} \right]^{3/2} e^{-\frac{3}{2} \left( \frac{u^2 + u_x^2 + u_y^2}{w^2} \right)}$$

where

$$w^2 = \frac{3kT}{m}$$

and \(T\) is the "temperature" corresponding to the energy of random motion of the plasma electrons.

Substitution into (1) and integration by expanding the denominator in a series results in the gain dispersion relation

$$1 = \frac{(\omega_b/\omega)^2}{(1-\nu_c/\omega)^2} + \frac{(\omega_b/\omega)^2}{1-\nu_c/\omega} \left[ 1 + \frac{\Gamma^2}{R\left[1-\nu_c/\omega\right]^2} + \frac{5\Gamma^4}{3R^2\left[1-\nu_c/\omega\right]^4} + \cdots \right]$$

where

$$\Gamma = \frac{\nu_c u_b}{\omega}$$

is a normalised propagation constant, and

$$R = \frac{\frac{2}{\nu_c}}{\frac{u_b^2}{w^2} = \frac{1}{2} \frac{m u_b^2}{kT}}$$

is the ratio of beam energy to average plasma electron energy.

For the series expansion to be valid, \(\Gamma^2/R\) must be much less than unity. The present experimental situation is such that the plasma
electrons have approximately four electron volts of random energy.

In order to investigate the gain versus frequency, Eq. (5) was solved, including only the first term of the expansion and assuming no collisions, on a digital computer. The equation then becomes

\[ 1 = \left( \frac{\omega_b}{\omega} \right)^2 + \left[ \frac{\omega_p}{\omega} \right]^2 \left( 1 + \frac{\Gamma^2}{R} \right) \]

or

\[ (\Gamma - 1)^2 (\Gamma^2 + \lambda) + \sigma = 0 \]

where

\[ \lambda = R \left[ 1 - \frac{\omega^2}{\omega_p^2} \right] \]

\[ \sigma = R \left[ \frac{\omega_b}{\omega_p} \right]^2 \]

For reasonable values of \( \sigma \) and \( R \), the growth constant is plotted in Fig. 2. Note that the horizontal coordinate is \( \omega/\omega_p \). The vertical coordinate is, of course, proportional to logarithm of gain.

\[ \sigma = R \frac{\omega_b^2}{\omega_p^2} \]

**Fig. 2** Theoretical growth constant versus normalized modulation frequency for several values of \( \omega_b/\omega_p \) and \( R \). The former is the ratio of beam electron density to the plasma electron density. The latter variable \( R \) is the ratio of beam electron energy to the average random energy of the plasma electrons.
To facilitate comparison with experimental data it would be convenient to replot power gain versus $(\omega_p/\omega)^2$. This will be discussed further in connection with the presentation of experimental results.

It should be recognized that the above analysis is purely one-dimensional and does not allow for the finite size of either the electron beam or the plasma column. The gain dispersion relation of Eq. (5) for plane waves in a one-dimensional plasma-beam geometry was originally derived from the Boltzmann equation. To proceed in the same manner allowing for the finite size of the electron beam and arc plasma column results in a more complicated partial differential equation for the perturbation in the distribution function. An alternative approach\(^6\) would be to work with the first few moments of the velocity distribution. This results in a set of differential equations which can be solved for growing wave propagation along a finite size electron beam and plasma column.

**Experimental Results**

A typical experimental tube is shown in Fig. 3. It differs only from the schematic of Fig. 1 in that the helices and their surrounding glass penetrates into the arc region so that the electron beam passes through a more uniform region of the plasma column than in Fig. 1.
The distance between the helices is still 5 cm. It should be emphasized that the electron beam is focused by positive ions and that no axial magnetic field is necessary, though a magnetic field has been applied with no qualitative change in the results.

At a given modulation frequency on the electron beam (typically 3000 Mc), the output signal is detected, filtered with a 1000 cps low-pass filter, and presented on an oscilloscope. When the arc current is swept at 50 cps, displays of power output versus arc current are obtained. Without filtering, the detected output is modulated with noise whose spectrum appears to be rather broad from 1 to 20 kc or so. This noise, which vanishes when the input signal is removed, may be associated with density fluctuations from moving striations.\(^7\)

For low pressure gaseous discharges, plasma charge density and hence \(\omega_p^2\) is directly proportional to arc current. Therefore the experimental photograph of Fig. 4(a) represents detected output signal vs. \(\omega_p^2\), since the horizontal axis is proportional to arc current. In this photograph the horizontal calibration is 5 divisions equals 1 ampere. The numerical solutions to the dispersion relation in Fig. 2 indicate that the maximum gain occurs when \(\omega\) is equal to \(\omega_p\) within a fraction of a percent, and, thus, the arc current of peak signal output in Fig. 4(a) effectively represents a precise measure of the plasma charge density on the axis of the arc discharge column. All the photographs of Fig. 4 were at a frequency of 3.0 kmc.

Curves similar to those of Fig. 4(a) are obtained at other frequencies and the arc current for maximum output is, as expected, found to vary as the square of the modulating frequency over the range 2.5 to 4.0 kmc. Such a plot of input modulation frequency squared versus arc current for maximum output signal is shown in Fig. 5,
curve $G$, for this growing wave experiment. The points represent actual experimental points taken from photographs such as Fig. 4(a). Curve $G$ in Fig. 5 forms a good straight line with only a small but finite intercept to the left of the origin, related to the contribution to the arc plasma density by the electron beam. This intercept has been removed by shifting the curve so as to pass through the origin. Thus experimental verification is obtained that the maximum rate of growth occurs when the plasma frequency is linearly related to the modulation frequency.

At the present time net gains between input and output waveguides of about 10 dB have been measured. With the arc plasma off but the electron beam on and optimally adjusted, the net loss through the tube is about 30 dB for typical values of beam current. This implies a gain of about 40 dB for the plasma-beam interaction region of 5 cm. This is, of course, only approximately true because the conditions of focusing and such on the electron beam are significantly affected by the arc being on or off. Attempts to measure the growth along the beam with a movable probe have been unsuccessful but will be repeated with a tube of smaller diameter. The estimated experimental growth constant along the plasma-beam interaction of 8 db per cm is significantly less than the theoretically predicted value as presented in Fig. 2 for $\sigma = 0.1$, $R = 100$, assuming no collisions, which predicts 42 db per cm. This large discrepancy is most likely
accounted for by slight inhomogeneities in the plasma density along the axis of the plasma region. Theoretical predictions of bandwidth of a fraction of a percent as seen from Fig. 2 imply that only slight inhomogeneities are needed to reduce the theoretical gain to the range of experimental values. In general, Fig. 4(a), which is power gain, agrees with Fig. 2, which is the logarithm of power gain, in that there is no growth until a certain value of \( \omega_p \) has been reached. Then a rapid increase of signal with plasma density occurs with a sharp maximum and then a less rapid fall-off of output signal as the plasma density is increased further.

While this growing wave experiment could be interpreted as a direct measurement of the electron density, it is essential in order to verify the theory that the electron density be measured by another method. This was accomplished by measuring the shift in resonant frequency of an S band cavity resonator caused by the plasma electrons. The results are shown by the circles marked C to the left in Fig. 5.

The same cavity was used to measure the plasma frequency in a long straight discharge tube. These results are shown by the circles marked C on the right in Fig. 5. The straight line curve labeled \( K \) represents data taken from a compilation of probe measurements by Klarfeld in mercury arc discharges, and gives an average \( \omega_p^2 \) versus \( I_A \) assuming a uniform plasma distribution throughout the cross section of the column.

Another measurement of electron density was made by observing the scattering of microwaves from the ionized column. The straight plasma column was inserted in a rectangular waveguide operating in the \( \text{TE}_{10} \) mode so that the axis of the column was perpendicular both to the electric field and to the direction of propagation, as in Fig. 6. When the reflection coefficient of the column is plotted versus arc current at a fixed frequency, two distinct maxima are found. Double peaks have been observed previously but not as yet explained. The elementary theory predicts only a single resonance at \( \omega_p^2/\omega^2 = 2 \) (modified by a straight loss correction for the glass walls of the column, giving \( \omega_p^2/\omega^2 = 2.81 \) in this case). It has been shown that reflection should occur approximately when the average square of the plasma frequency is equal to twice the square of the incident frequency \( \omega_p^2/\omega^2 = 2 \), unless the edge plasma density falls to less than about one-third of the axial density for an assumed parabolic variation of electron density with radius. In the latter case resonance occurs approximately when the edge plasma frequency is equal to the incident frequency. Figure 4(b) is a photograph of reflected energy versus arc current at 3.0 kmc. The horizontal calibration is 5 divisions equals 1 ampere. Two peaks in the reflected power are clearly visible. Also in evidence is hysteresis in the trace. This is probably due to non-equilibrium heating effects in the arc column. It is felt that the most meaningful data results
from the first occurring peaks of all the photographs in Fig. 4, since this display corresponds very closely with the results obtained when the arc current is varied very slowly. Such is plotted in Fig. 5.

The major resonance peak in Fig. 4(b) represents a reflection coefficient of nearly unity and is assumed to correspond to the elementary theory. The major peak curve plot in Fig. 5 is labeled $S_1$. The currents at resonance for the major peak have been divided by 2.81 so that the plot $S_1$ represents an estimate of $\omega_P^2$ versus arc current. The minor peak resonance current (curve $S_2$) is not divided by this reduction factor since the conditions for this resonance (i.e., at what value of $\omega_P^2/\omega^2$ the resonance occurs) are not known. Consequently, the curve $S_1$ lies to the left of $S_2$ in Fig. 5.

Theory predicts no resonance in scattering from an ionized column when the electric field is parallel to the axis of the column. Reflection was observed from a plasma column which was placed in a waveguide so that the electric field was parallel to the column, and the resulting reflected power is shown in Fig. 4(c) versus arc current. The vertical calibration is such that the reflection coefficient at 4 amperes (20 divisions) of arc current is approximately one-third. For increasing arc current the reflection coefficient continues to increase until the ionized column effectively represents a short across the waveguide. The purpose of this experiment was to investigate the possibility that the minor peak of resonance was caused by coupling to longitudinal oscillations. Such does not appear to be the case.

Another method of measuring the average plasma density of an ionized column is by utilizing a mode of wave propagation through a
stationary plasma column. This work has been reported previously by Gould and Trivelpiece.\textsuperscript{12,13} A slow wave is propagated down the ionized column when the incident frequency is less than the plasma frequency. A measure of the average plasma density is obtained by a low frequency phase velocity measurement. Utilizing such measurements, the average square of the plasma frequency is plotted versus arc current in curve W of Fig. 5.

\textbf{Conclusions}

Several new methods of measuring electron density have been presented. The relatively small discrepancies between the cavity, scattering, and wave propagation methods is probably accounted for by the radial density variation in the plasma together with experimental errors. The large difference between the results from the "bent" amplifier tube and the relatively long "straight" tube is accounted for by the pronounced difference in geometry.

It is felt that since the cavity, scattering and wave propagation methods tend to measure average or edge densities, whereas the growing-wave experiment measures axial density, this method of measurement with an electron beam has certain advantages due to its high spatial and frequency resolution. It is suggested that this mechanism may prove useful as a means of measuring radial variations in charge density in low-pressure gaseous discharges as well as other properties, such as characteristics of moving striations.

It should be pointed out that the scattering method cannot be used by itself to measure the average charge density unless it is known in some other way that the density does not vary greatly with radius. This method, when used in conjunction with a procedure which gives the plasma density on the axis of the column, does allow an estimate of the radial density variation.

\textit{Viewed as an amplifier, this experiment verifies the theoretical efforts of many workers by demonstrating the existence of growing plasma waves such as have been postulated as one of many possible sources of solar and other radio astronomical noise. Although present efforts have failed to produce gains quite as large as predicted by the rather simple one-dimensional theory, it is felt that the inclusion of statistical plasma inhomogeneities along the axis of the plasma column as well as the effects of collisions and finite geometry will account for the reduction in gain as well as the broadening of the frequency response curve. Certainly Fig. 4(a) demonstrates a sufficiently sharp response to be encouraging as to the validity of the theory.}
PLASMA INTERACTION EXPERIMENT

REFERENCES