

RESONANCE CONE STRUCTURE IN WARM ANISOTROPIC PLASMA *

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A fine structure is observed near the resonance cones in the field pattern of a small r.f. probe and is shown to result from an interference between a fast electromagnetic wave ($v_{\text{group}} \sim c$) and a slow plasma wave ($v_{\text{group}} \sim v_{\text{th}}$).

Experimental observations of resonance cones in the radio-frequency electric field pattern of a short probe in an anisotropic plasma have been reported and analyzed using cold plasma dielectric theory [1]. We report here observations of an *interference structure* near the cone angle which can be explained by the addition of electron thermal velocities to the theory.

Using Fourier transform methods we may solve for the potential, and hence the electric field near the resonance cone, of an oscillating charge $\rho_{\text{ext}} = q \exp(-i\omega t) \delta(r)$, yielding

$$\varphi = \frac{q \exp(-i\omega t)}{\epsilon_0} \iiint \frac{\exp(i\mathbf{k} \cdot \mathbf{r})}{k_{\parallel}^2 K_{\parallel} + k_{\perp}^2 K_{\perp}} \frac{k_{\perp} dk_{\perp} dk_{\parallel} d\phi}{(2\pi)^3} \quad (1)$$

For a cold plasma eq. (1) leads to our previous results [1]. The potential and fields are singular on the cone $\theta = \arctan \rho/z - \theta_c$ where $\tan^2 \theta_c = -K_{\perp}/K_{\parallel}$. For a warm plasma ($3kT_e = mv_{\text{th}}^2$), K_{\perp} and K_{\parallel} become functions of \mathbf{k} and eq. (1) can most easily be evaluated in the limit of a large static magnetic field ($\omega_c \gg \omega_p, \omega$), so that $K_{\perp} = 1$ and $K_{\parallel} = 1 - (\omega_p/k_{\parallel} v_{\text{th}})^2 Z'(\omega/k_{\parallel} v_{\text{th}})$. Performing the integrations over ϕ and k_{\perp}

$$\varphi(\rho, z) = \frac{q \exp(-i\omega t)}{4\pi^2 \epsilon_0} \int_{-\infty}^{\infty} K_0(k_{\parallel} \rho \sqrt{K_{\parallel}}) \exp(ik_{\parallel} z) dk_{\parallel} \quad (2)$$

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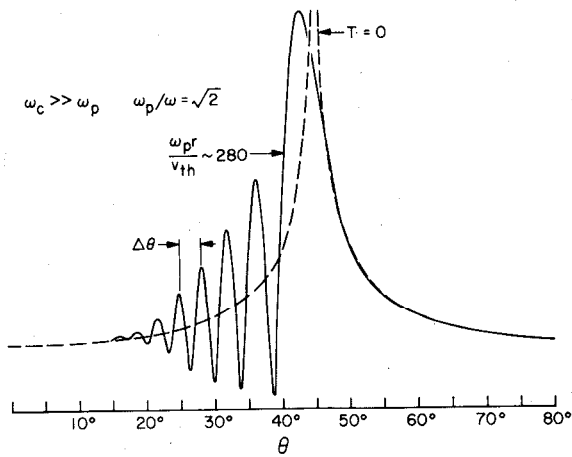


Fig. 1. Calculated angular dependence of the potential $|\varphi(\theta)|$ for $v_{th}/\omega_p r = 1/280$ (solid curve) and for $v_{th}/\omega_p r = 0$ (dashed curve).

where K_0 is the modified Bessel function. Numerical evaluation of eq. (2) yields the cone structure of fig. 1. The cone angle θ_c is shifted to a slightly smaller angle than that predicted by cold plasma theory ($\theta_c = \arcsin \omega/\omega_p$) and an interference structure appears inside the cone. An empirical fit to the numerical data or evaluation of eq. (2) by the method of stationary phase yields an angular interference spacing $\Delta\theta \propto (v_{th}/\omega_p r)^{2/3}$ for constant θ_c , where r is the probe separation. A calculation of the fields of an oscillating dipole based on the full set of Maxwell's equations [2] yields a similar relationship for the interference spacing. For $\omega < \omega_p$ two waves of different phase velocity exist, whose group velocities make a given angle $\theta < \theta_c$ with respect to the static magnetic field [3]. The observed structure arises from the interference of these two waves.

An experimental study of the cone structure was performed in the argon discharge previously described [1] with the static magnetic field kept as large as possible ($\omega_c \approx 2.3 \omega$) to best approximate the limit $B_0 \rightarrow \infty$. The experimentally observed interference structure † was not as pronounced as in fig. 1, but the amplitude of the interference structure depends on the relative

† Fig. 1 of ref. 1 exhibits this structure, although it is not discussed.

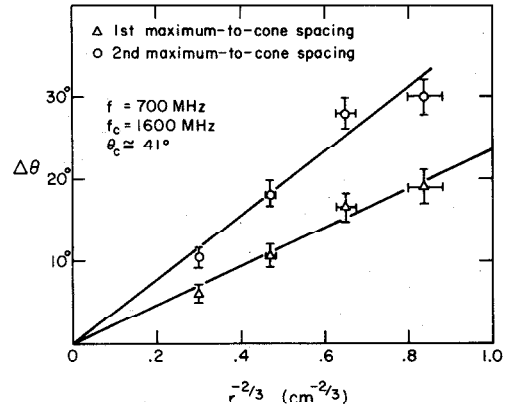


Fig. 2. Experimental interference spacing $\Delta\theta$ versus $r^{-2/3}$ (r = probe separation).

amplitude of the slow and fast waves, which in turn depends on the type of probe exciting the waves. The angular spacing is, however, dependent only on the relative phase of the slow and fast waves and does not depend on the type of probe used.

Comparing the experimental spacing $\Delta\theta$ to that calculated using the warm plasma theory outlined above, we infer that $T_e = 2-3$ eV. A Langmuir probe measurement yields $T_e \approx 3$ eV.

To further test the theory we constructed four receiving probes, each at a different radius from the transmitting probe in order to verify that $\Delta\theta \propto r^{-2/3}$. In fig. 2 the triangular points are the experimental spacing between the first maximum of the interference structure and the cone; the circular points, that between the second maximum and the cone. Both sets of data are consistent with the $r^{-2/3}$ dependence. Similar data, taken under a variety of plasma conditions, when plotted on a log-log plot and fitted to a straight line by a least squares procedure, yields an average slope -0.74 ± 0.05 . This somewhat higher value than the expected $-\frac{2}{3}$ may be due to perturbations of the plasma conditions around the probes.

References

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3. R. W. Gould and R. K. Fisher, Proc. 9th Intern. Conf. on Phenomena in ionized gases, Bucharest, Romania, 1969, p. 435.

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